# INDAM workshop on
# “Irregular Stochastic Analysis”

Cortona, 23-27 June 2025

## About

The INDAM workshop **Irregular Stochastic Analysis** will be held at Palazzone di Cortona (Italy), on 23-27 June 2025.

Website: `https://eventi.unibo.it/irreg-stoch-indam-cortona-25`

## Aims and Scope

The scope of this workshop is to promote the integration of approaches and techniques developed by different communities that share a common interest in irregular stochastic analysis, to gain deep insight into a variety of problems related to stochastic models arising from specific applications that include the following:

- McKean-Vlasov equations and related Fokker-Planck equations, including kinetic models;
- Models for spike trains of large neuronal systems driven by stochastic point processes;
- Stochastic dynamical systems and their limiting stochastic PDEs;
- Multi-player stochastic control problems and mean-field games;
- Complex systems, self-organized criticality, porous media equations;
- Entropic optimization and transport.

The workshop also aims at fostering the interaction between recognized experts in the field and early career researchers.

## Organizing committee

For any questions, contact one of the organisers:
Elena Issoglio (University of Torino) at elena.issoglio@unito.it
Stefano Pagliarani (University of Bologna) at stefano.pagliarani9@unibo.it
Francesco Russo (ENSTA Paris) at francesco.russo@ensta.fr

## Sponsors

The conference is sponsored by *Istituto Nazionale di Alta Matematica* - INDAM, with the co-sponsorship of

- the *Department of Mathematics of the University of Bologna*;
- the *ANR SDAIM*;
- the *Department of Mathematics of the University of Torino*;

## Useful Information

All scientific activities will take place in Palazzone di Cortona, 52044 Le Contesse (AR).
Talks will be in the room "Sala Papacello", equipped with a projector and a blackboard.

The Social Dinner on Thursday 26th will be served at Osteria del Teatro, Via Giuseppe Maffei, 2 - 52044 Cortona (AR).

## Programme

This is a preliminary programme, it may be subject to changes.

| **Cortona - 23-27 June 2025 - Programme** | | | | | |
|---|---|---|---|---|---|
| | | | | | |
| | **Monday** | **Tuesday** | **Wednesday** | **Thursday** | **Friday** |
| 08:45 - 09:00 | opening | | | | |
| 09:00 - 09:50 | Henderson | Röckner | Flandoli | Delarue | Cerrai |
| 09:50 - 10:10 | chair (Pagliarani) | chair (Barbu) | chair (Delarue) | chair (Caravenna) | chair (Dalang) |
| 10:10 - 10:45 | Ottobre | Djurdjevac | Olivera | Menozzi | Marino |
| 10:45 - 11:15 | coffee break | coffee break | coffee break | coffee break | coffee break |
| 11:15 - 11:50 | Ohashi | Tölle | Kohatsu-Higa [11:15 - 12:05] | Dalang | Shamarova |
| 11:50 - 12:25 | Rüdiger | Ciotir | chair (Tölle) [12:05 - 12:25] | Di Girolami | Richard |
| | | | | | |
| 12:25 - 14:00 | lunch | lunch | lunch | lunch | lunch & closing |
| | | | | | |
| 14:00 - 14:50 | Barbu | Caravenna | de Bouard | Di Nunno | |
| 14:50 - 15:10 | chair (Menozzi) | chair (Flandoli) | chair (Rüdiger) | chair (Kohatsu-Higa) | |
| 15:10 - 15:45 | Ferrario | Simon | Masiero | Pascucci | |
| 15:45 - 16:15 | coffee break | coffee break | coffee break | coffee break | |
| 16:15 - 16:25 | | Amato | Rondelli | | |
| 16:25 - 16:35 | | Cagnotti | Ventura | | |
| 16:35 - 16:45 | | Dehò | Tarquini | | |
| 16:45 - 16:55 | | Rui | Arnese | | |
| | | | | | |
| 19:30 - 22:30 | | | | social dinner | |

# Titles and Abstracts

### Numerical approximation of McKean-Vlasov SDEs via stochastic gradient descent

Andrea Amato

We propose a novel approach to numerically approximate McKean-Vlasov stochastic differential equations (MV-SDE) using stochastic gradient descent (SGD) while avoiding the use of interacting particle systems. The technique of SGD is deployed to solve a Euclidean minimization problem, which is obtained by first representing the MV-SDE as a minimization problem over the set of continuous functions of time, and then by approximating the domain with a finitedimensional subspace. Convergence is established by proving certain intermediate stability and moment estimates of the relevant stochastic processes (in- cluding the tangent ones). Numerical experiments illustrate the competitive performance of our SGD based method compared to the IPS benchmarks. This work offers a theoretical foundation for using the SGD method in the context of numerical approximation of MV-SDEs, and provides analytical tools to study its stability and convergence.
Based on a joint work with A. Agarwall, G. Dos Reis and S. Pagliarani in [1]

**References**

1. A. Agarwal, A. Amato, G. d. Reis, and S. Pagliarani. Numerical approxima- tion of mckean-vlasov sdes via stochastic gradient descent. arXiv preprint arXiv:2310.13579, 2023.

### Regularity and Propagation of Chaos for Conditional McKean–Vlasov Equations

Manuel Arnese

We study the rate of propagation of chaos for a McKean–Vlasov equation with conditional expectation terms in the drift. We use a (regularized) Nadaraya–Watson estimator at a particle level to approximate the conditional expectations; we then combine relative entropy methods in the spirit of Jabin and Wang with information theoretic inequalities to obtain the result. The non-parametric nature of the problem requires higher regularity for the density of the McKean–Vlasov limit, which we obtain with a bootstrap argument and energy estimates.

### Fokker-Planck equations with integral drifts and McKean-Vlasov SDEs

Viorel Barbu

The existence and uniqueness of weak solutions to nonlinear Fokker-Planck equations with singular integral drifts and the existence of strong solutions to corresponding McKean-Vlasov SDEs is discussed.

### $L^p$ convergence of a numerical scheme for SDEs with distributional coefficients

Matteo Cagnotti

This talk addresses the extension of convergence results for a numerical scheme for one-dimensional stochastic differential equations with drift coefficients that are Hölder continuous in time and belonging to a negative order Besov space in space as first established by Jáquez, Issoglio, and Palczewski. They prove that a two-step scheme converges with rate $1/6$ in the best case in a strong $L^1$ sense.
By using estimates previously used in the literature to prove convergence in $L^p$ in the case of SDEs with Hölder continuous coefficients in both variables, the results for distributional drifts can be extended to the $L^p$ norm for $p \geq 2$. Although the rate deteriorates as $p$ increases, it remains positive for all finite $p$.

### Noise sensitivity for 2d Stochastic Heat Equation and directed polymers

Francesco Caravenna

We consider the (ill-defined) 2d Stochastic Heat Equation with multiplicative space-time white noise. Upon discretisation of space-time, the solution coincides with the partition function of 2d directed polymers in random environment: under a critical (logarithmic) rescaling of the noise strength, it converges to a universal limit known as the critical 2d Stochastic Heat Flow.
We prove that discretised solution (or partition function) is noise sensitive, i.e. any small perturbation of the underlying noise produces a solution which becomes asymptotically independent of the original one. This is obtained by generalising classical criteria for noise sensitivity beyond the boolean setting, which have an independent interest. As a corollary, the Stochastic Heat Flow is shown to be independent of the white noise driving the corresponding Stochastic Heat Equation.
(Based on joint work with Anna Donadini)

### Parabolic rescaling of a stochastic wave map: limit and fluctuations

Sandra Cerrai

I consider a wave map equation in dimension 1+1 perturbed by a noise of co-normal type. After applying a parabolic rescaling, I show that the position converges to the solution of a deterministic heat flow that retains a memory of the noise. I then analyze the fluctuations around this deterministic limit and prove a "weak" central limit theorem. As a byproduct of this analysis, I also derive several results for the deterministic wave map and heat flow equations that appear not to be contained in the previous literature.

### A Stochastic Stefan Problem With Mushy Region and Turbulent Transport Noise

Ioana Ciotir

This work is devoted to the proof of the existence of a martingale solution for a complex version of the stochastic Stefan problem. This particular formulation incorporates two important features: a mushy region and turbulent transport within the liquid phase. While our approach bears similarities to porous media equations, it differs in a crucial aspect. Instead of using the typical framework for such equations, we have chosen to work within an $L^2$ space. This choice is motivated by the nature of the operator that characterizes the turbulent noise in our model. The $L^2$ space provides a more natural and appropriate setting for handling this specific operator, allowing us to better capture and analyze the turbulent transport phenomena in the liquid phase of the Stefan problem. The last part of the work establishes a scaling limit theorem for the Stefan problem incorporating a mushy region on a torus, demonstrating that solutions to stochastic variants with turbulent transport terms converge to the solution to a deterministic partial differential equation. (joint work with Franco Flandoli and Dan Goreac)

### Sharp upper bounds on hitting probabilities for the solution to the stochastic heat equation on the line

Robert Dalang

For Gaussian random fields with values in $\mathbb{R}^d$, sharp upper and lower bounds on the probability of hitting a fixed set have been available for many years. These apply in particular to the solutions of systems of linear SPDEs. For non-Gaussian random fields, the available bounds are less sharp. For systems of stochastic heat equations, a sharp lower bound was obtained in [1]. Here, we obtain the corresponding sharp upper bound. This is based on joint work with Fei Pu and David Nualart.

[1] R.C. Dalang and F. Pu, Optimal lower bounds on hitting probabilities for stochastic heat equations in spatial dimension $k \geq 1$. Electron. J. Probab. 25 (2020), Paper No. 40, 31 pp.

## The Gibbs measure of the renormalized two dimensional stochastic Gross-Pitaevskii equation

Anne de Bouard

The stochastic Gross-Pitaevskii equation is a mean-field model designed to describe a Bose-Einstein condensate close to the critical condensation temperature. It is a complex Ginzburg-Landau equation, with a harmonic confining potential and additive space-time white noise. In this talk, we will discuss the space dimension-two case, for which renormalization is required. We will construct the Gibbs measure for this equation, which is also formally invariant for the nonlinear Schrödinger equation with harmonic potential in dimension two. It will also be shown that this measure is singular with respect to the underlying Gaussian measure.
This talk is based on joint works with A. Debussche (ENS Rennes) and R. Fukuizumi (Waseda University, Japan).

## Rotational invariance of integration by parts formula and Lie symmetries of SDEs

Susanna Dehò

The study of symmetry properties of ordinary differential equations and partial differential equations is a classical and well-established topic in the literature, offering both a powerful tool for the explicit computation of solutions and a deeper understanding of their qualitative behavior. In contrast, a theory of symmetries for stochastic differential equations (SDEs), analogous to the deterministic case, has been developed only in recent years ([1],[2],[3]), also highlighting significant connections with the symmetries of the associated Fokker-Planck or Kolmogorov equations (see [4], [5], [6]). The application of Lie symmetry theory to SDEs enables the derivation of integration by parts formulas inspired by Bismut's variational approach to Malliavin calculus [9], with notable applications to the analysis of the law and regularity of the processes, as well as to the development of a stochastic calculus of variations (see [7], [8]). Various notions of invariance properties and symmetries for SDEs, including strong, weak, and gauge symmetries, will be described here, with particular emphasis on the rotational invariance of the driving Brownian motion and the associated infinitesimal generator. The stochastic rotational invariance of the integration by parts formula proposed in [9] will also be demonstrated and discussed through applications to selected Brownian motion-driven stochastic models ([10]).
This poster is based on a joint work with F.C. De Vecchi, P. Morando and S. Ugolini.

**References:**

1. Lazaro-Cami, Joan-Andreu and Ortega, Juan-Pablo, Reduction, reconstruction, and skew-product decomposition of symmetric stochastic differential equations, Stochastics and Dynamics, 9, (2009) 1-46
2. G. Gaeta, C. Lunini, and F. Spadaro, Recent advances in symmetry of stochastic differential equations, Rend. Mat. Appl.(7), 39, (2018) 293–306
3. S. Albeverio, F. C. De Vecchi, P. Morando, and S. Ugolini, Random transformations and invariance of semimartingales on Lie groups, Random operators and Stochastic Equations, (2021).
4. F. C. De Vecchi, P. Morando, and S. Ugolini, Symmetries of stochastic differential equations using

Girsanov transformations, Journal of Physics A: Mathematical and Theoretical, 53 (2020), no. 13, 135204.
5. R. Kozlov, Symmetries of Kolmogorov backward equation, Journal of Nonlinear Mathematical Physics, 28 (2021), 182–193.
6. G.Gaeta, and M. A. Rodriguez Integrable Ito equations and properties of the associated Fokker-Planck equations, Open Communications in Nonlinear Math ematical Physics, 3,(2023).
7. J.-M. Bismut, Martingales, the Malliavin calculus and hypoellipticity under general Hormander's conditions, Zeitschrift fur Wahrscheinlichkeitstheorie und verwandte Gebiete, 56 (1981), no. 4, 469–505.
8. V. I. Bogachev, Differentiable Measures and the Malliavin Calculus, Mathematical Surveys and Monographs, vol. 164, American Mathematical Society, Providence, RI, 2010.
9. F. C. De Vecchi, P. Morando, and S. Ugolini, Integration by parts formulas and Lie's symmetries of SDEs, Electronic Journal of Probability, vol. 30 (2025).
10. S. Dehò, F. C. De Vecchi, P. Morando, and S. Ugolini, Random rotational invariance within a Bismut-type approach to integration by parts formulas, Preprint 2025.

## Rearranged Stochastic Heat Equation

François Delarue

The rearranged stochastic heat equation is a version of the 1d stochastic heat equation that is reflected on the boundary of quantile functions on the torus by means of a rearrangement procedure. By isometry between the space of quantile functions equipped with $L^2$ distance and the space of probability measures (on $\mathbb{R}$) equipped with the 2-Wasserstein distance, it provides a diffusion process on the space of 1d probability measures. In a recent work with W. Hammersley (just appeared in PTRF), we explored the construction and the properties of this process. In an going work with R. Likibi, we explore the situation when the dynamics on the space of probability measures are also subjected to a convolution operation: at infinitesimal time step, the probability valued process moves according to the rearranged dynamics and then is convoluted by an infinitesimal Gaussian kernel. The latter convolution accounts for the impact of idiosyncratic noises in large particle systems, whist the rearranged dynamics would correspond to the impact of an infinite dimensional common noise.

## Dam Management in the Era of Climate Change

Cristina Di Girolami

Climate change has a dramatic impact, particularly by concentrating rainfall into a few short periods, interspersed by long dry spells. In this context, the role of dams is crucial. We consider the optimal control of a dam, where the water level must not exceed a designated safety threshold, nor fall below a minimum level to ensure functionality and sustainability for for the outgoing river. To model dry spells and intense rainfall events, commonly referred to as water bombs, we introduce a Hawkes process, a well-known example of a self-exciting process characterised by time-correlated intensity, which endogenously reproduces the concentration of events. The problem is formulated as an optimal switching problem with constraints. We establish existence results and propose numerical methods for approximating the solution. Finally, we illustrate the main achievements of this approach through numerical examples. The main and counterintuitive result of our numerical analysis is that the optimal water level inside the dam increases with the self-exciting parameter. This result shows that, when facing the dilemma of managing the opposing risks of dam overtopping and dry spells, the former ultimately dominates the latter. In conclusion, dams will increasingly lose their role as water reserves and take on a greater role in flood protection.

### Stochastic optimal control for quadratic hedging in presence of memory

Giulia Di Nunno

Sandwiched Volterra Volatility (SVV) models are a class of models able to capture both the long memory and the rough aspects of volatility as well as complying with many of the stylised features typical of volatilities. We present the model and its properties. We then discuss questions of quadratic hedging. Particularly, while the theoretical solution is characterised in terms of the non-anticipating derivative for all square integrable claims, the fact that these models are typically non-Markovian provides a concrete difficulty in the direct computation of conditional expectations at the core of the explicit hedging strategy. To overcome this difficulty, we propose a Markovian approximation of the model which stems from an adequate approximation of the kernel in the Volterra noise.
The presentation is based on joint work with Anton Yurchenko-Tytarenko (UiO, Statkraft)

### Nonlinear SPDE models of particle systems

Ana Djurdjevac

Interacting particle systems provide flexible and powerful models that are useful in many application areas such as sociology (agents), molecular dynamics (proteins) etc. However, particle systems with large numbers of particles are very complex and difficult to handle, both analytically and computationally. Therefore, a common strategy is to derive effective equations that describe the time evolution of the empirical particle density. A prototypical example that we will consider is the formal identification of a finite system of particles with the singular Dean-Kawasaki equation. We will give a short introduction about the Dean-Kawasaki equation and its applications. Our aim is to introduce a well-behaved nonlinear SPDE that approximates the Dean- Kawasaki equation for a particle system with mean-field interaction both in the drift and the noise term. We want to study the well-posedness of these nonlinear SPDE models and to control the weak error of the SPDE approximation with respect to the particle system using the technique of transport equations on the space of probability measures. This is the joint work with H. Kremp, N. Perkowski and J. Xiaohao. Furthermore, we will discuss possible numerical methods for these problems.

### The nonlinear Schrödinger equation with multiplicative noise and arbitrary power of the nonlinearity

Benedetta Ferrario

We consider the nonlinear Schrödinger equation on the d-dimensional torus, with the nonlinearity of polynomial type $|u|^{2\sigma}u$. For any $\sigma \in \mathbb{N}$ and $s > d/2$ we prove that adding to this equation a suitable stochastic forcing term there exists a unique global solution for any initial data in $H^s$. The effect of the noise is to prevent blow-up in finite time, differently from the deterministic setting. Moreover, we prove the existence of an invariant measure and its uniqueness under more restrictive assumptions on the noise term.
This is based on a joint work with Z. Brzezniak, M. Maurelli and M. Zanella.

### Point vortices and their links to PDEs

Franco Flandoli

We revise some known results about point vortices under a new perspective and also for modified models (with respect to the classical 2D Euler euqations), like the so-called Hasegawa-Mima equation. One of the links is the approximation of point vortices by smooth solutions; the main one we investigate is the approximation of solutions by point vortices, in the framework of McKean-Vlasov equations. We give a new proof of the second approximation result, based on the first one of the approximations mentioned above and a weak-strong uniqueness property. We also discuss a number of open questions. The activities mentioned herein are performed in the framework of the project NoisyFluid (GA 101053472).

### Nash inequalities and boundary behavior of kinetic equations

Christopher Henderson

Kinetic equations model systems, such as a gas, where particles move through space according to a velocity that is diffusing (due to, say, collisions with other particles). The presence of spatial boundaries in these models causes technical issues because they are first order in the spatial variable and therefore cannot be defined everywhere on the boundary. In this talk, I will present $L^1 - L^\infty$ estimates that yield sharp bounds on the behavior at the spatial boundary. The main estimate is a kinetic version of the Nash inequality.
This is a joint work with Giacomo Lucertini and Weinan Wang.

### A general framework for an approximation method for invariant measures of stochastic equations

Arturo Kohatsu Higa

We propose a general framework to implement an Euler like approximation method for stochastic equations under the Wasserstein distance. The framework originates from explicit results for the diffusion case due to Lamberton and Pages, Pages and Panloup between others. We believe that the proposed general framework applies to many examples. In particular, we show that it applies to Langevin type equations, reflected equations, Boltzmann type stochastic equations and some neuronal models. This framework is built using some basic ideas from the sewing lemma.
This is ongoing joint work with V. Bally (U. G. Eiffel) and A. Alfonsi (ENPC).

### Super-diffusive limit for a kinetic interface model

Lorenzo Marino

We study the long time behaviour of the solutions to a linearized Boltzmann equation in one spatial dimension, subject to a random mechanism of transmission, reflection or absorption/generation at an interface. Assuming a fast enough degeneracy of the scattering kernel and a slow decay for the probability of absorption at low frequencies, we show that the solutions to the interface model exhibit a super-diffusive behaviour in the long time limit, with the scaling parameter depending only on the interplay between the decay velocities of the scattering kernel and the drift. We also characterise such a limit as the unique weak solution to a non-local in space evolution equation, subject to a reflection-transmission condition at the interface. Our proof relies on a merger between probabilistic techniques, exploiting that the kinetic dynamics away from the interface is indeed the Kolmogorov equation for a classic jump process, and analytical results on Dirichlet forms.
This talk is based on a work in collaboration with Tomasz Komorowski (Polish Academy of Sciences) and K Bogdan (Wroclaw University of Science and Technology).
The activity was carried out within the project: NoisyFluid "Noise in Fluids", Grant Agreement 101053472, CUP E53C22001720006.

**Bismut-Elworthy type formulae for BSDEs with degenerate noise**

Federica Masiero

In this talk we present how to derive a Bismut- Elworthy formula under assumptions weaker than non degeneracy of the noise. We can apply our results to stochastic (damped) wave equations, whose regularizing properties of the transition semigroup are discussed.
The talk is based on joint works with D. Addona and E. Priola.

**Weak error for Numerical schemes associated with SDEs with singular drifts**

Stephane Menozzi

We will review some recent results concerning the numerical approximation of SDEs with singular drift driven by an additive rotationally invariant alpha-stable process. We will focus on the error on the densities for drifts which can belong to Hölder, Lebesgue or Besov spaces. Some open related problems will be discussed as well.

**Solving non-Markovian stochastic control problems driven by Wiener functionals**

Alberto Ohashi

In this talk, we will present a general methodology for stochastic control problems driven by the Brownian motion filtration including non-Markovian and non-semimartingale state processes controlled by mutually singular measures. The general convergence of the method is established under rather weak conditions for distinct types of non-Markovian and non-semimartingale states. Explicit rates of convergence are provided in case the control acts only on the drift component of the controlled system. Near-closed/open-loop optimal controls are fully characterized by a dynamic programming algorithm and they are classified according to the strength of the possibly underlying non-Markovian memory. The theory is applied to stochastic control problems based on path-dependent SDEs and rough stochastic volatility models, where both drift and possibly degenerated diffusion components are controlled. Optimal control of drifts for nonlinear path-dependent SDEs driven by fractional Brownian motion with exponent $H \in (0, 1/2)$ is also discussed.

**Quantitative approximation of the Navier-Stokes-Vlasov-Fokker-Planck System by stochastic particles systems**

Christian Olivera

This talk is concerned with a fluid-particle system. More precisely, we consider the incompressible Navier-Stokes equations coupled to the Vlasov equation through the drag force. This model arises from the research of aerosols, sprays or more generically two-phase flows. We derive the rate of convergence of a system of particles, interacting with a fluid, to Navier-Stokes-Vlasov-Fokker-Planck system. The empirical measure of particles is proved to converge to the Vlasov-Fokker-Planck component of the system and the velocity of the fluid coupled with the particles converges to the Navier-Stokes component.

**Interacting particle systems, McKean-Vlasov PDEs and S(P)DEs with additive noise**

Michela Ottobre

When dealing with systems which are made of a large number of particles, one is often interested in the collective behaviour of the system rather than in a detailed description. Established approaches in statistical mechanics and kinetic theory allow one to study the limit as the number of particles N tends to infinity and to obtain a (low dimensional) PDE for the evolution of the density of particles. The limiting PDE is a non-linear equation, where the non-linearity has a specific structure and is called a McKean-Vlasov nonlinearity. Of course one of the issues here is which properties of the initial particle systems are preserved upon passing to the limit – and this is something we will touch upon. Even if the particles evolve according to a stochastic differential equation, the limiting equation is deterministic, as long as the particles are subject to independent sources of noise. If the particles are subject to the same noise (common noise) then the limit is given by a Stochastic Partial Differential Equation (SPDE). In the latter case the limiting SPDE is substantially the McKean-Vlasov PDE + noise; noise is further more multiplicative and has gradient structure. One may then ask the question about whether it is possible to obtain McKean-Vlasov SPDEs with additive noise from particle systems. We will explain how to address this question, by studying limits of weighted particle systems. We will moreover discuss applications of the problem of sampling from the invariant distribution of SPDEs with additive noise.

### Well-posedness and propagation of chaos for kinetic McKean-Vlasov equations

Andrea Pascucci

We consider a McKean-Vlasov system of kinetic type, where the equation is degenerate because the dimension of the driving Brownian motion is strictly smaller than that of the solution, as typically occurs in classical models of collisional kinetic theory. Assuming Hölder continuous coefficients and a weak Hörmander condition, we prove the well-posedness of the equation. Under stronger assumptions, we also show that the system propagates chaos.

### Regularity of the density of SDEs driven by fractional noise and application to McKean-Vlasov equations

Alexandre Richard

First, we will consider the SDE $dX_t = b(t, X_t)dt + dB_t$, where $b$ is a singular drift (e.g. a distribution) and $B$ is a fractional Brownian motion. We will review some recent results on existence and uniqueness for this equation, providing criteria linking the regularity of $b$ and the Hurst parameter $H$ of the fractional Brownian motion. Next, we will study the time-space regularity of the conditional density of the solution in Lebesgue-Besov spaces, and also provide Gaussian bounds. Then by exploiting this regularity, we will demonstrate the existence of solutions for McKean-Vlasov equations of the form $dY_t = \mu_t * b(t, Y_t) + dB_t$, where $\mu_t$ is the law of the solution $Y_t$, for a drift $b$ that can be more singular than in the linear case, and chosen in the full sub-critical regime of such SDEs. Finally, we discuss uniqueness for this singular McKean-Vlasov equation.
Joint work with L. Anzeletti, L. Galeati and E. Tanré. Abstract

### Nonlinear Markov processes in the sense of McKean

Michael Röckner

In the talk we shall discuss nonlinear Markov processes in the sense of McKean's seminal work in PNAS 1966. In particular, we shall present a general new technique how to show that a family of probability measures on cadlag paths, given by the path laws of solutions to a McKean-Vlasov type SDE, form a

nonlinear Markov process. The SDE's coefficients are only assumed to be measurable in their measure variable, so that they may depend on derivatives of any order of the solutions' time-marginal densities. In particular, the p-Brownian motion associated to the parabolic p-Laplace equation turns out to be a nonlinear Markov process in the sense of McKean. Further examples are related to the generalized (fractional) porous media equation, the Burgers and the 2D vorticity Navier-Stokes equation.
Joint work with Marco Rehmeier

### Existence and uniqueness results for strongly degenerate McKean-Vlasov equations with rough coefficients

Alessio Rondelli

We study existence results for weak solutions to a broad class of degenerate McKean- Vlasov equations with rough coefficients. These are Stochastic Differential Equations where the coefficients also depend upon the flow of marginals of the solution. Strong well-posedness has been classically proved for Lipschitz coefficients of linear growth; in this talk we present a weak existence and uniqueness result for rough coefficients and degenerate diffusion. In particular we will use Krylov's estimates and Girsanov's theorem to tackle the case with linear dependence on the marginal.

### Existence and identification of Boltzmann processes

Barbara Rüdiger

The stochastic differential equation of McKean-Vlasov type is identified such that the Fokker-Planck equation associated to it is the Boltzmann equation. Hence, we call its solutions as Boltzmann processes. They describe the dynamics (in position and velocity) of particles expanding in vacuum in accordance with the Boltzmann equation. Given a solution $f := f(t, x, v)_{[0,T]}$ of the Boltzmann equation, the existence of solutions to the McKean-Vlasov SDE is established for the hard sphere case. This is a joint work with P. Sundar (Louisiana State University)

### Stochastic Interacting Particles with Strongly Singular Repulsion Coupled to a Continuum Environment with Killing

Giulia Rui

We consider a system of stochastic interacting particles subject to strong repulsion and reactive killing, coupled with an evolving continuum environment through a feedback mechanism. The particles follow stochastic differential equations with singular pairwise interaction drift, derived from a generalized Lennard-Jones potential, and a non-local term modeling interaction with the surrounding environment. Particles may be removed from the system according to an inhomogeneous Poisson process, representing local chemical reactions. The environment field evolves according to the law of mass action, and therefore depends on the empirical distribution of active particles, introducing a feedback between stochastic and continuous dynamics.

We prove the well-posedness of the coupled system, despite the presence of singular interactions. In a simplified setting where the continuum field is externally prescribed and memoryless, we establish a first propagation of chaos result via a relative entropy approach. This framework serves as a foundation for future analytic developments and rigorous scaling limits in interacting particle systems coupled to evolving environments.

### Singular SPDEs with the Cauchy-Riemann operator on a torus

Evelina Shamarova

We prove the existence of solution to the following $\mathbb{C}^3$-valued singular SPDE on the 2D torus $\mathbb{T}^2$:

$$\partial_{\bar{z}} r = r \times \bar{r} + i\,\gamma\,\mathscr{W},$$

where $\partial_{\bar{z}} := \frac{1}{2}(\partial_x + i\partial_y)$ is the Cauchy-Riemann operator on $\mathbb{T}^2$, $\mathscr{W} = (\mathscr{W}_1, \mathscr{W}_2, \mathscr{W}_3)$ is a real 3D white noise on $\mathbb{T}^2$ whose component $\mathscr{W}_3$ has zero mean over $\mathbb{T}^2$, $\gamma := (\gamma_1, \gamma_2, \gamma_3)$ is an $\mathbb{R}^3$-vector and $\gamma\mathscr{W} := (\gamma_1\mathscr{W}_1, \gamma_2\mathscr{W}_2, \gamma_3\mathscr{W}_3)$.
This talk is based on a joint work with Zdzislaw Brzezniak and Mikhail Neklyudov.

### Macroscopic evolution of mechanical and thermal energy in a harmonic chain

Marielle Simon

It has been recently observed that some physical or biological systems which are maintained in a bath of constant temperature can behave in an unexpected way: in some cases the temperature stationary profile presents a maximum inside the system higher than the thermostats temperatures, as well as the possibility of uphill diffusion (energy current against the temperature gradient). This is the case for instance in mitochondria, which are present in nearly all types of human cell. In a collaborative work with T. Komorowski and S. Olla, we derive rigorously this "heating inside the system" phenomenon from a microscopic infinite chain of coupled oscillators in contact at both ends with heat baths at different temperatures, and subject to an external force at one end. While heat flows from the thermostats, the mechanical energy produced by the force is then transformed into thermal energy by the bulk dynamics. We also obtain, with only one thermostat to the left, the convergence of the two conserved quantities towards the solution of a non-linear diffusive system of conservative partial differential equations with a Dirichlet type and Neumann boundary conditions on the left and the right endpoints, respectively.

### Probabilistic interpretation of McKean PDEs in the non-conservative and path-dependent case

Leonardo Tarquini

We present two probabilistic interpretations of a nonlinear parabolic partial differential equation (PDE) with path-dependent coefficients, by deriving a McKean-Vlasov-type stochastic differential equation (SDE) whose time-marginal densities solve the given PDE. In both approaches, we begin by considering a regularized version of the PDE. In the first approach, we associate to this regularized PDE a McKean-Vlasov SDE coupled with a Feynman-Kac-type equation. We then construct the corresponding particle system and establish propagation of chaos.
In the second, alternative framework, we introduce an SDE with McKean-Vlasov-type drift and a killing mechanism. Here, we analyse a moderately interacting particle system that evolves via diffusion until a stopping time determined by its empirical density. Using an analytical semigroup approach, we establish convergence of the empirical density to the solution of the original, non-regularised PDE. This study is motivated by applications: the PDE under consideration models the sulphation phenomenon—a degradation process affecting marble surfaces exposed to atmospheric pollutants.
This is joint work with D. Morale and S. Ugolini (University of Milano).

### Ergodicity and mixing for locally monotone stochastic evolution equations with Lévy noise

Jonas Tölle

We establish general quantitative conditions for stochastic evolution equations with locally monotone drift and degenerate additive Lévy noise in variational formulation resulting in the existence of a unique invariant probability measure for the associated ergodic Markovian Feller semigroup. We prove improved moment estimates for the solutions and the $e$-property of the semigroup. Furthermore, we provide quantitative upper bounds for the Markovian $\epsilon$-mixing times. Examples include the stochastic incompressible 2D Navier-Stokes equations, shear thickening stochastic power-law fluid equations, the stochastic heat equation, as well as, stochastic semilinear equations such as the 1D stochastic Burgers equation.

### Wasserstein Distance in Terms of the Comonotonicity Copula

Irene Ventura

We focus on reformulating the $p$-Wasserstein distance $W_p$, defined with the $p$-norm for $p \in [1, \infty)$ on $\mathbb{R}^d$, in terms of copulas. First, we consider the one-dimensional case ($d = 1$) and prove that the comonotonicity copula yields the optimal coupling for the $p$-Wasserstein distance $W_p$. We then extend this result to the multivariate case ($d > 1$), under the sufficient and necessary condition that the distributions involved share the same copula. Moreover, we analyze a particular variant of the Wasserstein distance, $W_{p,q}$, as defined by Alfonsi and Jourdain (2014), for $p, q \in [1, \infty)$. We provide upper and lower bounds for $W_{p,q}$ in terms of $W_p$, and consequently, in terms of the comonotonicity copula. Finally, we extend our analysis to the countermonotonicity copula, proving that similar considerations hold in this case as well.
This is joint work with Mariem Abdellatif, Peter Kuchling, and Barbara Rüdiger.